EELE 461/561 – Digital System Design

Module #7 – Lossy Lines

- **Topics**
  1. Skin Effect
  2. Dielectric Loss

- **Textbook Reading Assignments**
  1. 9.1-9.11

- **What you should be able to do after this module**
  1. Describe the physical phenomenon behind Skin Effect and Dielectric Loss
  2. Use a modern CAD tool to simulate the behavior of lossy transmission lines

Lossy Transmission Lines

- **Lossy Lines**

- When we derived Telegrapher's Equations, we made an assumption that there was no loss in the equivalent circuit model (i.e., R=0, G=0)

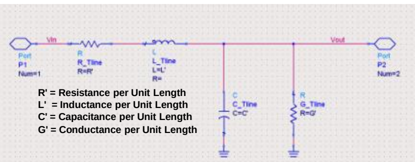
- This allowed us to simplify the math and come up with the following important equations for a **Lossless T-line:**

$$Z_0 = \sqrt{\frac{L}{C}} \qquad T_D = \sqrt{LC}$$

Lossy Transmission Lines

- **Circuit Model**

- If the line is lossy, we need to include the series resistance and shunt conductance back into our equivalent T-line circuit model



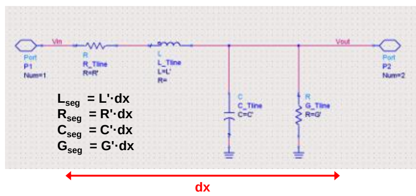
Lossy Transmission Lines

- **Circuit Model**

- Let's derive the relationship between Voltage & Current to time and space with the full model

- We first define the *length* of the wire using dx.

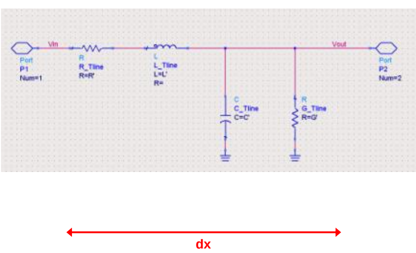
- Since our electrical components are defined in *unit length*, the total values can be found by multiplying the *unit length* value by *dx*.



Lossy Transmission Lines

- **Circuit Model**

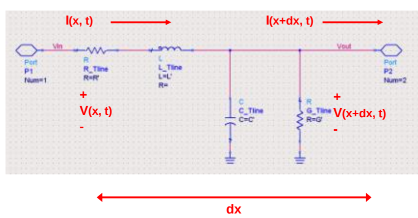
- We enter the segment at (*x*) and we exit the line at *(x+dx)*



Lossy Transmission Lines

- **Circuit Model**

- The input voltage can be described as: V(x,t)
- The input current can be described as: I(x,t)
- The output voltage can be described as: V(x+dx,t)
- The output current can be described as: I(x+dx,t)



Lossy Transmission Lines

- **Circuit Model**

  - We can write an expression for the voltage drop across the inductor and resistor using KVL:

$$V(x,t)-V(x+dx,t)=R'dx\cdot I(x,t)+L'dx\cdot\frac{dI(x,t)}{dt}$$

  - Which is rewritten as:

$$-\left[\frac{V(x+dx,t)-V(x,t)}{dx}\right]=R'\cdot I(x,t)+L'\cdot\frac{dI(x,t)}{dt}$$

  - If we let dx→0, we are left with a differential equation:

$$-\frac{dV(x,t)}{dx}=R'\cdot I(x,t)+L'\cdot\frac{dI(x,t)}{dt}$$

Lossy Transmission Lines

- **Circuit Model**

  - Now we can write an expression for the output current using KCL:

$$I(x,t)-I(x+dx,t)=G'dx\cdot V(x+dx,t)+C'dx\cdot\frac{dV(x+dx,t)}{dt}$$

  - Which can be rewritten as:

$$-\left[\frac{I(x+dx,t)-I(x,t)}{dx}\right]=G'\cdot V(x+dx,t)+C'\cdot\frac{dV(x+dx,t)}{dt}$$

  - If we let dx→0, we are left with another differential equation:

$$-\frac{dI(x,t)}{dx}=G'\cdot V(x,t)+C'\cdot\frac{dV(x,t)}{dt}$$

Lossy Transmission Lines

- **Circuit Model**

  - These two 1st order differential equations describe the complete interaction of V and I on a T-line. (i.e., "***Telegrapher's Equations")***

$$\boxed{-\frac{dV}{dx}=R'\cdot I+L'\cdot\frac{dI}{dt}\qquad -\frac{dI}{dx}=G'\cdot V+C'\cdot\frac{dV}{dt}}$$

  - Let's put these into a more usable form using phasor representation where (d/dt → jω)

$$\boxed{-\frac{dV(x)}{dx}=(R'+j\omega L')\cdot I(x)\qquad -\frac{dI(x)}{dx}=(G'+j\omega C')\cdot V(x)}$$

Lossy Transmission Lines

- **Wave Equations**

  - We can put the voltage expression into the form of the Wave Equation by differentiating the first *Telegrapher* equation with respect to *x*

differentiate with respect to x

$$-\frac{dV(x)}{dx}=(R'+j\omega L')\cdot I(x)$$

$$\downarrow$$

$$-\frac{d^2V(x)}{dx^2}=(R'+j\omega L')\cdot\frac{dI(x)}{dx}$$

Lossy Transmission Lines

- **Wave Equations**

  - We can now plug in our expression for the derivative of current:

$$-\frac{d^2V(x)}{dx^2}=(R'+j\omega L')\cdot\frac{dI(x)}{dx}\quad\leftarrow\quad\boxed{-\frac{dI(x)}{dx}=(G'+j\omega C')\cdot V(x)}$$

$$-\frac{d^2V(x)}{dx^2}=-(R'+j\omega L')\cdot(G'+j\omega C')\cdot V(x)$$

  - Rearranging, we get:

$$\frac{d^2V(x)}{dx^2}-(R'+j\omega L')\cdot(G'+j\omega C')\cdot V(x)=0$$

Lossy Transmission Lines

- **Wave Equation**

  - If we define g as:

$$\gamma=\sqrt{(R'+j\omega L')\cdot(G'+j\omega C')}$$

  - we can rewrite the voltage expression in Wave Equation form as:

$$\frac{d^2V(x)}{dx^2}-\gamma^2\cdot V(x)=0$$

Lossy Transmission Lines

- **Wave Equations**

- Now let's put the current expression into the form of the Wave Equation by differentiating the second *Telegrapher* equation with respect to *x*

differentiate with respect to x

$$-\frac{dI(x)}{dx} = (G' + j\omega C') \cdot V(x)$$

$$\downarrow$$

$$-\frac{d^2I(x)}{dx^2} = (G' + j\omega C') \cdot \frac{dV(x)}{dx}$$

Lossy Transmission Lines

- **Wave Equations**

- We can now plug in our expression for the derivative of voltage:

$$-\frac{d^2I(x)}{dx^2} = (G' + j\omega C') \cdot \frac{dV(x)}{dx} \qquad \boxed{-\frac{dV(x)}{dx} = (R' + j\omega L') \cdot I(x)}$$

$$-\frac{d^2I(x)}{dx^2} = -(G' + j\omega C') \cdot (R' + j\omega L') \cdot I(x)$$

- Rearranging, we get:

$$\frac{d^2I(x)}{dx^2} - (R' + j\omega L') \cdot (G' + j\omega C') \cdot I(x) = 0$$

Lossy Transmission Lines

- **Wave Equations**

- Since, we have already defined g as:

$$\gamma = \sqrt{(R' + j\omega L') \cdot (G' + j\omega C')}$$

- we can rewrite the current expression in Wave Equation form as:

$$\frac{d^2I(x)}{dx^2} - \gamma^2 \cdot I(x) = 0$$

Lossy Transmission Lines

- **Wave Equations**

- These two 2nd order differential equations describe the voltage and current propagation on a lossy transmission line:

$$\boxed{\frac{d^2V(x)}{dx^2} - \gamma^2 \cdot V(x) = 0 \qquad \frac{d^2I(x)}{dx^2} - \gamma^2 \cdot I(x) = 0}$$

- We call g the "Complex Propagation Constant"

$$\gamma = \sqrt{(R' + j\omega L') \cdot (G' + j\omega C')}$$

which consists of a *Real* and *Imaginary* part:

$$\gamma = \alpha + j\beta$$

Lossy Transmission Lines

- **Attenuation Constant**

- We call the Real part of g the *Attenuation Constant (α)*

$$\alpha = \mathrm{Re}(\gamma) = \mathrm{Re}\left(\sqrt{(R' + j\omega L') \cdot (G' + j\omega C')}\right)$$

- This quantity has units of Np/m

NOTE: A Napier is unit of *ratio*. It is used to describe ratios of voltages and currents.

$$Np = \ln\left(\frac{x_1}{x_2}\right)$$

- Remember that in a *Lossless* line, α=0.

- In a *Lossy* line $\alpha \neq 0$

Lossy Transmission Lines

- **Phase Constant**

- We call the Imaginary part of g the *Phase Constant (β)*

$$\beta = \mathrm{Im}(\gamma) = \mathrm{Im}\left(\sqrt{(R' + j\omega L') \cdot (G' + j\omega C')}\right)$$

- This quantity has units of rad/m

- This can be used to calculate the wave velocity with:

$$velocity = \mu_p = \frac{\omega}{\beta}$$

- **Complex Propagation Constant for Passive T-lines**

- We choose the square root values of g that give positive values for α and β.

$$\gamma = \alpha + j\beta$$

- For passive T-lines,

$$\alpha \geq 0$$

Lossy Transmission Lines

- **Characteristic Impedance**

  - The Wave Equations have traveling wave solutions for a lossy medium in the form of:

$$V(x) = V^{+}e^{-\gamma x} + V^{-}e^{+\gamma x} = V^{+}(x) + V^{-}(x)$$
$$I(x) = I^{+}e^{-\gamma x} + I^{-}e^{+\gamma x} = I^{+}(x) + I^{-}(x)$$

Forward Traveling Wave

Reverse Traveling Wave

Lossy Transmission Lines

- **Characteristic Impedance**

  - If we differentiate the voltage solution, we can get the solution for current by plugging it back into our original Telegrapher's Equation:

$$V(x) = V^{+}e^{-\gamma x} + V^{-}e^{+\gamma x}$$

$$\frac{dV}{dx} = -\gamma V^{+}e^{-\lambda x} + \gamma V^{-}e^{+\gamma x} = -(R' + j\omega L') \cdot I(x)$$

$$\frac{dV}{dx} = \gamma\left[V^{+}e^{-\gamma x} - V^{-}e^{+\gamma x}\right] = (R' + j\omega L') \cdot I(x)$$

  - Rearranging for I, we get:

$$I(x) = \frac{\gamma}{R' + j\omega L'} \cdot \left[V^{+}e^{-\gamma x} - V^{-}e^{+\gamma x}\right]$$

Lossy Transmission Lines

- **Characteristic Impedance**

  - Remembering the definition for impedance is the ratio of *either* the forward or reverse traveling waves:

$$Z = \frac{V^{+}}{I^{+}} = \frac{-V^{-}}{I^{-}}$$

  - We can now plug in our equations for V(x) and I(x), and only consider the forwarding traveling waves:

$$Z_0 = \frac{V}{I} = \frac{\left[V^{+}e^{-\gamma x} + V^{-}e^{+\gamma x}\right]}{\frac{\gamma}{R' + j\omega L'} \cdot \left[V^{+}e^{-\gamma x} - V^{-}e^{+\gamma x}\right]} = \frac{\left[V^{+}e^{-\gamma x}\right]}{\frac{\gamma}{R' + j\omega L'} \cdot \left[V^{+}e^{-\gamma x}\right]}$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \frac{R' + j\omega L'}{\sqrt{(R' + j\omega L') \cdot (G' + j\omega C')}} = \frac{\sqrt{R' + j\omega L'} \cdot \sqrt{R' + j\omega L'}}{\sqrt{(R' + j\omega L')} \cdot \sqrt{(G' + j\omega C')}}$$

$$Z_0 = \frac{\sqrt{R' + j\omega L'}}{\sqrt{(G' + j\omega C')}} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

Lossy Transmission Lines

- **Characteristic Impedance**

  - Now we have our expression of the characteristic impedance of a lossy transmission line:

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

  - Notice that this quantity is complex. This implies that the magnitude of the impedance depends on the incidence waves frequency.

  - When a digital signal is traveling in a lossy line, each harmonic component of the square wave will experience a different impedance.

Sources of Loss

- **Sources of Loss**

  - *Loss* refers to the amount of signal transmitted that does not reach the receiver.

  - Loss can occur due to many sources:

    1) Impedance Mismatches Leading to Reflected Energy
    2) Coupling to adjacent Traces
    3) Radiation Loss
    4) Conductor Loss
    5) Dielectric Loss

  - When we talk about *Lossy Transmission Lines*, we tend to focus on items 4 & 5

  - Reflections & Coupling can be modeled using our standard LC model of a T-line.

  - Radiation is typically small, and is difficult to model using circuit models so we treat that using EM simulators.

  - Conductor Loss and Dielectric Loss can be modeled using our complete RLCG transmission line circuit.

  - When people talk about *Lossy Lines,* they are typically referring to Conductor & Dielectric Loss

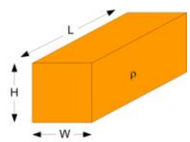
Conductor Loss

- **Skin Depth**

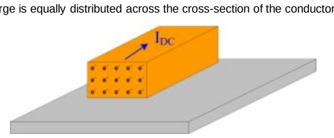
  - At DC, the resistance of a conductor is proportional to:

    - the cross-sectional area of the conductor
    - the bulk resistivity of the material

$$R = \frac{\rho \cdot L}{W \cdot H} = \frac{\rho \cdot L}{Area}$$



  - At DC, the charge is equally distributed across the cross-section of the conductor:



Conductor Loss

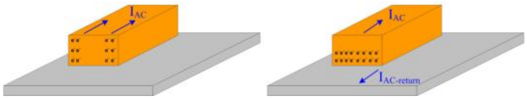
- **Skin Depth**

  - When AC current flows through the conductor, the charge is not equally distributed within the cross-sectional area of the conductor. At AC, the current will attempt to find the path of <u>least impedance</u>

  - This results in two trends:

    1) The current within the conductor will spread out as far as possible in order to minimize the partial self-inductance of the conductor:

    2) The current within the conductor will try to move as close as possible to the oppositely directed return current in more to maximum the partial mutual inductance between the two currents.



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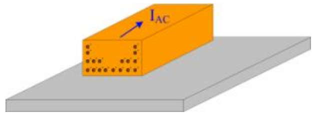
Conductor Loss

- **Skin Depth**

  - The phenomenon of the current flowing through this *reduced cross-section* of the conductor is described using a quantity called <u>**Skin Depth**</u>

  - The reduced cross-section has the effect of increasing the series resistance of the conductor as the frequency increases.



Conductor Loss

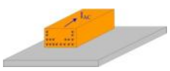
- **Skin Depth**

  - Skin Depth is described by δ and is expressed as:

$$\delta = \sqrt{\frac{2\cdot\rho}{\omega\cdot\mu}} = \sqrt{\frac{\rho}{\pi\cdot\mu\cdot f}} = \frac{1}{\sqrt{\pi\cdot\mu\cdot f\cdot\sigma}}$$

where: ρ is the resistivity of the conductor
σ is the conductivity of the conductor (1/ ρ)
μ is the magnetic permeability ($\mu_0 \cdot \mu_r$)

  - Skin depth has units of *meters* and is the definition of the depth below the surface of a conductor with depth (d) at which the current density (J) decays to 1/e (~0.37) of the current density of the surface (J_S): $J = J_S e^{-d/\delta}$



Conductor Loss

- **Skin Depth**

  - Skin Depth directly effects the series resistance of a Transmission line by reducing the effective cross-sectional area of the conductor that the current can flow through.

$$R_S = \frac{\rho\cdot L}{Area(\delta)}$$

  - Notice that the skin depth is inversely proportional to the square root of frequency:

$$\delta = \frac{1}{\sqrt{\pi\cdot\mu\cdot f\cdot\sigma}} \propto \frac{1}{\sqrt{f}}$$

  - Since skin depth is in the denominator of the expression for series resistance of the conductor, this means that the resistance is proportional to the square root of frequency:

$$R_S \propto \sqrt{f}$$

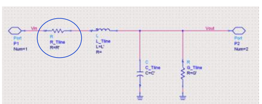
Conductor Loss

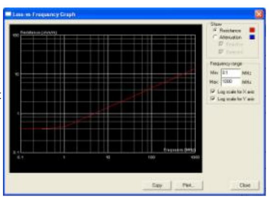
- **Skin Depth**

  - We model the conductor loss due to skin depth using the series resistor in our RLGC T-line model.

  - This resistance value is dependant on frequency:

$$R_S(f) \propto \sqrt{f}$$





  - When stimulating the lossy transmission line with a digital signal, each frequency component of the signal will experience a different series resistance.

Conductor Loss

- **Total Conductor Loss**

  - The complete expression for conductor loss should also include any DC loss due to the resistivity of the bulk material:

$$R_{conductor} = R_{DC} + R_{AC}\sqrt{f}$$

  - Note that it is hard to define one complete expression for the AC conductor resistance because it depends on the geometry of the conductor. (i.e., round, square, rectangle, etc…)

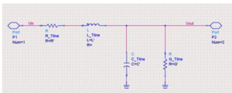
  - For a given cross sectional shape, the skin depth is then applied to that shape in order to predict the new cross-sectional area that the current flows through.

Lossy Transmission Lines

- **Dielectric Loss**

  - Now we move to the 2nd main effect in lossy lines, the dielectric.

  - We use the G' element in our model to account for dielectric loss.



Lossy Transmission Lines

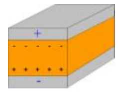
- **Dielectric Loss**

  - An ideal capacitor will store a particular amount of charge depending on the voltage applied.

$$C = \frac{Q}{V}$$

  - The capacitor is constructed using two conducting surfaces separated by a dielectric.

  - In the ideal case, this structure does NOT allow DC current to flow between the two terminals of the device.



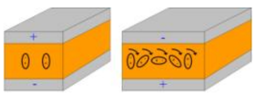
Lossy Transmission Lines

- **Dielectric Loss**

  - In reality, the charge in the capacitor is held by electric dipoles within the material.

  - If an AC voltage is applied to the capacitor, the dipoles will align to the direction of the applied electric field.

  - This *movement* of charge results in AC current flow.



  - The amount of current that flows is proportional to the rate-of-change of voltage across the capacitor:

$$i_c = C\frac{dV}{dt}$$

Lossy Transmission Lines

- **Dielectric Loss**

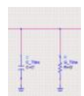
  - Since there is no DC current (or no in-phase current), there is no real power dissipation in an ideal capacitor.

  - This allows us to say that the resistivity of an ideal capacitor is ∞.

  - However, real capacitors do have some resistance associated with them.

  - This means that current will flow through the capacitor that is *in phase* with the voltage.

  - We model this resistance with a resistor component in parallel with the capacitor.



Lossy Transmission Lines

- **Dielectric Loss**

  - The density of available dipoles in a material to hold the charge is reflected in the dielectric constant (i.e., the electric permittivity).

  - The density of dipoles available in air is described with a permittivity of ε_0

  - If a parallel plate capacitor was constructed using an air dielectric, the capacitance would be given by:

$$C_0 = \frac{\varepsilon_0 \cdot A}{t}$$

  - If the air was then replaced with a different insulating material with ε_r>1, the new capacitance would be described as:

$$C = \varepsilon_r \cdot C_0$$

Lossy Transmission Lines

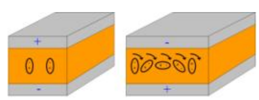
- **Dielectric Loss**

  - The dipoles in the material do not re-orient instantaneously upon a change in voltage.

  - For a given time varying voltage, there are going to be dipoles that are perfectly aligned with the applied electric field. These dipoles contribute to the capacitance of the structure and result in an ***out of phase current (-90)***

  - Due to the finite speed at which the dipoles can change, there will be dipoles that are aligned perpendicular to the applied electric field which produce an ***in phase current***

  - This ***in phase current*** is the source of the leakage current in a capacitor



Lossy Transmission Lines

- **Dielectric Loss**

  - If we apply an arbitrary sinusoidal voltage to a capacitor:

$$V = V_0 \cdot e^{j\omega t}$$

the current that results is described as:

$$I = C\frac{dV}{dt} = j\omega CV$$

- We can use our definition for a non-air dielectric capacitor to include the dielectric constant in this equation:

$$I = C\frac{dV}{dt} = j\omega \varepsilon_r C_0 V$$

Lossy Transmission Lines

- **The Complex Dielectric Constant**

  - We now can use the relative permittivity of the material to describe both the ***in phase current*** and the ***out of phase current*** by making it a complex quantity:

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$$

where:

ε_r = the complex dielectric constant

ε_r' = the real part of the complex dielectric constant, representing the ***out of phase current*** flow in the capacitor. This is what we have always called simply *the dielectric constant*. Remember that this current is always defined as being -90º out of phase with the voltage

ε_r'' = the imaginary part of the complex dielectric constant, representing the ***in phase current*** flow in the capacitor. This is the new quantity that we've added to represent the current due to the loss in the dielectric. Since this current is in phase, we need to apply a "-" sign in the complex quantity to re-align it back to the voltage (remember by definition the current in the capacitor starts at -90º)

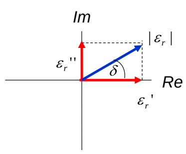
Lossy Transmission Lines

- **The loss angle (δ)**

  - we describe the angle between the real and imaginary parts of the complex dielectric constant as the *loss angle* **(δ)**

  - Note: this is NOT the same as the skin depth, it is just an unfortunate coincidence that delta is used for both. Sometimes people will use θ for the *loss angle*

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$$



Lossy Transmission Lines

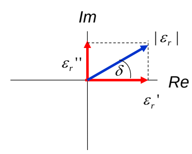
- **The Loss Tangent, (tan(δ))**

  - By convention, we use the ratio of the imaginary to the real parts of the dielectric constant to describe the loss in a material:

  - This is called the ***Loss Tangent*** **(δ)** or **Dissipation Factor**

$$\tan(\delta) = \frac{\varepsilon_r''}{\varepsilon_r'}$$

$$\varepsilon_r'' = \varepsilon_r' \cdot \tan(\delta)$$



Lossy Transmission Lines

- **The Loss Tangent (Dissipation Factor)**

  - At this point, things can get a little confusing. Remember that:

    - the dielectric constant is described as a complex quantity in order to describe both:

      - the ***out of phase current*** due to an applied voltage

      - the ***in phase current*** due to an applied voltage

    - The *out of phase* current that results from an applied voltage is actually the expected response of an ideal capacitor. That is why this quantity is described using the *real* part of the complex dielectric constant.

    - The *in phase* current that results from an applied voltage is due to the loss in a real capacitor and is described using the *imaginary* part of the complex dielectric constant.

Lossy Transmission Lines

- **The Loss Tangent (Dissipation Factor)**

  - Now let's describe the loss in the dielectric using the Loss Tangent.

  - The complete expression for the current in our capacitor is given by:

$$I = j\omega\varepsilon_r C_0 V = j\omega\left(\varepsilon_r^{'} - j\varepsilon_r^{''}\right)C_0 V = j\omega\varepsilon_r^{'} C_0 V + \omega\varepsilon_r^{''} C_0 V = \omega\varepsilon_r^{''} C_0 V + j\omega\varepsilon_r^{'} C_0 V$$

$$I(\mathrm{Re}) = \omega\varepsilon_r^{''} C_0 V$$

$$I(\mathrm{Im}) = \omega\varepsilon_r^{'} C_0 V$$

  - The resistance of the material is described as:

$$R_{leakage} = \frac{V}{I(\mathrm{Re})} = \frac{V}{\omega\varepsilon_r^{''} C_0 V} = \frac{1}{\omega\varepsilon_r^{''} C_0} = \frac{1}{\omega\varepsilon_r^{'}\tan(\delta) C_0} = \frac{1}{\omega\tan(\delta) C}$$

Lossy Transmission Lines

- **The Loss Tangent (Dissipation Factor)**

- We can now put his expression for $R_{leakage}$ into terms of bulk conductivity of the material:

$$R_{leakage} = \frac{1}{\omega \tan(\delta) C}$$

- The definition of resistance & capacitance of a test structure is (looking from the top to bottom):

$$R = \frac{\rho \cdot Length}{Area} = \frac{1}{\sigma} \cdot \frac{H}{Area} \qquad C = \frac{\varepsilon_0 \cdot \varepsilon_r' \cdot Area}{H} = \varepsilon_0 \cdot \varepsilon_r' \cdot \frac{Area}{H}$$

- These two expressions can be combined to form a relationship between the loss tangent & the bulk conductivity of the material:

$$R = \frac{1}{\sigma} \cdot \frac{H}{Area} = \frac{1}{\sigma} \cdot \frac{\varepsilon_0 \varepsilon_r'}{C} = \frac{1}{\omega \tan(\delta) C}$$

$$\sigma = \varepsilon_0 \varepsilon_r' \omega \tan(\delta)$$

Lossy Transmission Lines

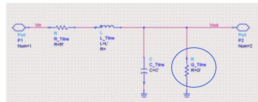
- **The Loss Tangent (Dissipation Factor)**

- Now we have the bulk conductivity (and resistivity) in terms of the Loss Tangent and frequency:

$$\sigma = \varepsilon_0 \varepsilon_r' \omega \tan(\delta)$$

$$\rho = \frac{1}{\varepsilon_0 \varepsilon_r' \omega \tan(\delta)}$$

- This can we used to describe the conductance or resistance of the shunt resistor in our model:



$$G' \propto f$$

$$R' \propto \frac{1}{f}$$

- We typically use G' to model the dielectric loss because it scales directly with length. That way all 4 of our T-line parameters will scale with length

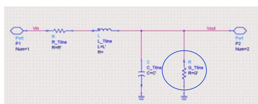
Lossy Transmission Lines

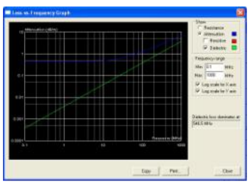
- **Dielectric Loss**

- We model the conductor loss due to skin depth using the shunt conductance in our RLGC T-line model.

- This conductance value is dependant on frequency:

$$G_{shunt}(f) \propto f$$





- When stimulating the lossy transmission line with a digital signal, each frequency component of the signal will experience a different shunt conductance.

Lossy Transmission Lines

- **Total Loss**

- we now see that there are two frequency dependant sources of loss in our lossy T-line.

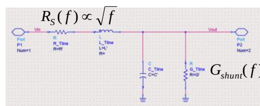
- This results in higher frequencies being attenuated more than lower frequencies.

- The two frequency dependant sources are spec'd in terms of *unit length* (i.e., R', G')

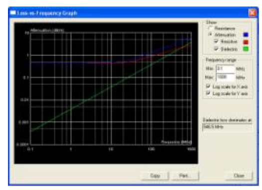
$$G_{dielectric} = 2\pi \cdot f \cdot \tan(\delta) C$$

$$R_S = R_{DC} + R_{AC}\sqrt{f}$$

$$R_S(f) \propto \sqrt{f}$$



$$G_{shunt}(f) \propto f$$



Lossy Transmission Lines

- **Signal Velocity & Dispersion**

- we have seen that the velocity of a sine wave in a lossy medium depends on the frequency.

$$u_p = \frac{\omega}{\beta}$$

- For a digital signal, this means that different frequency components will travel through the lossy line at different rates.

- The end result will be a distorted signal at the receiver.

- This phenomenon is called ***Dispersion***

Lossy Transmission Lines

- **Approximations**

- There are a couple approximations that can be made to simplify the analysis of lossy lines:

Low Loss Regime

- when $R_S << X_L$ and $G_L << X_C$

- we can use the lossless equations for Z_0 and velocity

Good Conductor

- when $R_S << X_L$ (i.e., skin effect is negligible)

$$\alpha = \sqrt{\pi f \mu \sigma} \qquad \beta = \sqrt{\pi f \mu \sigma}$$

Low Loss Dielectric

- when $G_L << X_C$ (i.e, the dielectric leakage is negligible)

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \qquad \beta = \omega\sqrt{\mu\varepsilon}$$