EELE 250: Circuits, Devices, and Motors

Lecture 11

Assignment Reminder

- Read 4.1 - 4.3 AND 5.1 – 5.4
- Practice problems:
  - P3.46, P3.54, P3.62, P3.63
  - P4.3, P4.5, P4.33, P4.39
- D2L Quiz #5 by 11AM on Monday 3 Oct.
- REMINDER: Work on your Lab #3 formal report. The reports are due at lab time during the week of Oct. 3. Lab #4 will be performed this week—be sure to do the pre-lab assignment calculations!

Sinusoidal Current and Voltage

- $v(t) = V_m \cos(\omega t + \theta)$

- $\omega = 2 \pi f$ [radians / sec]
- f = frequency [cycles / sec] or [Hz]
- T = 1 / f = period [sec]

- Root mean square (RMS) concept

Sinusoids

- Which is the correct relationship between sine and cosine?
    A. $\cos(\theta) = \sin(\theta + \pi/2)$
    B. $\cos(\theta) = \sin(\theta - \pi/2)$
    C. $\cos(\theta) = \sin(\theta + \pi)$
    D. $\cos(\theta) = \sin(\theta - \pi)$
    E. $\cos(\theta) = -\sin(\theta)$
- (answer is A)

Sinusoids

- Which is the correct relationship between sine and cosine?
    A. $\sin(\theta) = -\cos(\theta)$
    B. $\sin(\theta) = \cos(\theta + \pi/2)$
    C. $\sin(\theta) = \cos(\theta - \pi)$
    D. $\sin(\theta) = \cos(\theta - \pi/2)$
    E. $\sin(\theta) = \cos(\theta + \pi)$
- (answer is D)

Phasors

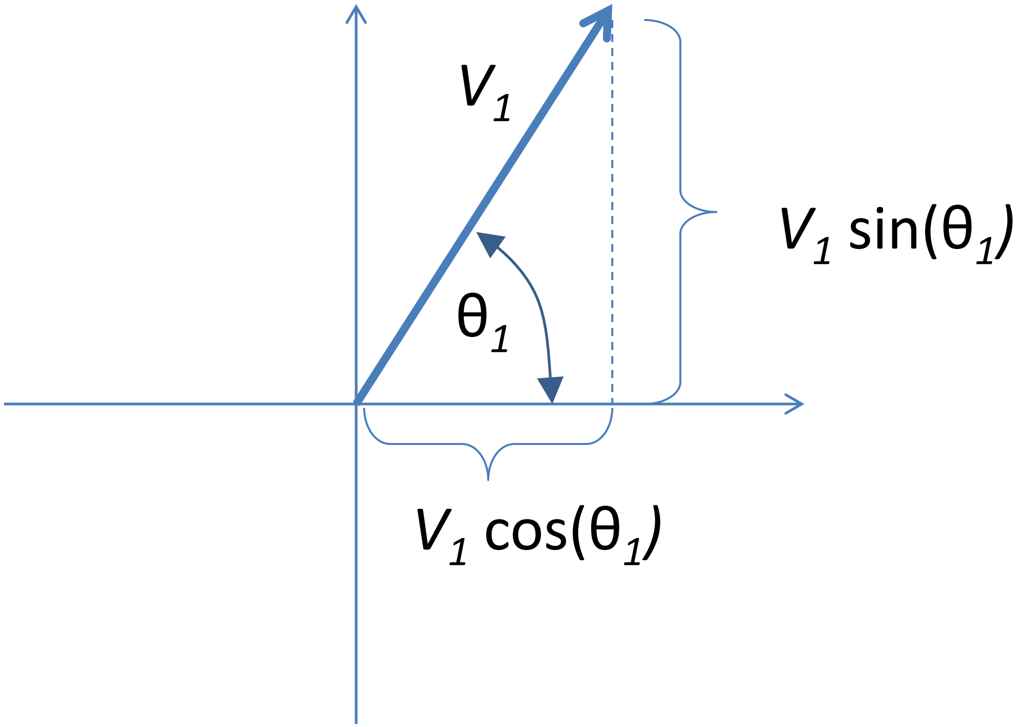
- Represent a sinusoid $v(t) = V_1 \cos(\omega t + \theta_1)$ as a vector of length V_1 and angle θ_1 with respect to the real axis
- This vector is equivalent to a *complex number*

  real part is $V_1 \cos(\theta_1)$

  and

  imaginary part is $V_1 \sin(\theta_1)$
- (Polar form vs. rectangular form)

Phasors (cont.)



Phasors (cont.)

- Circuits with sinusoidal signals often result in KVL or KCL expressions like:

$$V_1 \cos(\omega t + \theta_1) + V_2 \cos(\omega t + \theta_2) + V_3 \cos(\omega t + \theta_3)$$

It is a pain to add these signals via trigonometric identities!

Fortunately, it is easier to add using phasors: add the vectors as complex numbers.

Phasors (cont.)

$V_1 \cos(\omega t + \theta_1) + V_2 \cos(\omega t + \theta_2) + V_3 \cos(\omega t + \theta_3)$

Phasors: $V_1 \angle\theta_1 + V_2 \angle\theta_2 + V_3 \angle\theta_3$

Real parts: $V_1 \cos(\theta_1) + V_2 \cos(\theta_2) + V_3 \cos(\theta_3)$

Imag parts: $V_1 \sin(\theta_1) + V_2 \sin(\theta_2) + V_3 \sin(\theta_3)$

Sum phasor:

sqrt(real^2 + imag^2) $\angle$atan(imag/real)

Complex impedances

- Inductor: $v(t)$ = L di/dt
- If $i(t) = I_m \cos(\omega t)$, then $v(t) = -\omega\, I_m\, L \sin(\omega t)$

  ⇨ *note that* $-\sin(\omega t) = \cos(\omega t + 90^\circ)$

- As phasors:

  $\boldsymbol{I} = I_m \angle 0^\circ$ $\qquad\qquad$ $\boldsymbol{V} = \omega\, I_m\, L \angle 90^\circ$

which means:

$$\boldsymbol{V} = (\ \omega\, L \angle 90^\circ\ ) \cdot (\boldsymbol{I})$$

Note: ω L $\angle 90^\circ$ is the complex number $j\, \omega$ L

Complex Impedances (cont.)

- $\boldsymbol{V} = (\omega \, L \angle 90°) \cdot (\boldsymbol{I}) = (j \, \omega \, L) \cdot (\boldsymbol{I})$
- Ohm's Law: $V = I \cdot R$, can be generalized to

  $\boldsymbol{V} = \boldsymbol{I} \cdot \boldsymbol{Z}$, where $\boldsymbol{Z}$ is the *impedance*.
- $\boldsymbol{Z}$ can be a real or a complex number
  - Impedance of a resistor: $\mathbf{Z} = R$
  - Impedance of an inductor: $\mathbf{Z} = j \, \omega \, L$
  - Impedance of a capacitor: $\mathbf{Z} = 1/(j \, \omega \, C)$

Complex Impedances (cont.)

- NOTE that the impedance of an inductor or capacitor depends upon the sinusoidal frequency, ω

$$Z_L = j\omega L \qquad Z_C = \frac{1}{j\omega C}$$

- Impedance magnitude of inductor goes *up* as frequency increases
- Impedance magnitude of capacitor goes *down* as frequency increases

Summary and Review

- Represent a group of sinusoids with the same frequency as *phasors*
- Add phasors by interpreting them as complex numbers
- Generalize Ohm's Law to be **V** = **I** **Z**
- Impedance of a resistor: **Z** = R
- Impedance of an inductor: **Z** = j ω L
- Impedance of a capacitor: **Z** = 1/(j ω C)