EELE 577 Mid-term Practice Exam #1

Open book and notes. *Four Problems, Three Pages*

Part I. A causal LTI system is defined by the z-transform

$$H(z) = \frac{10z^{-1}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1}\right)}.$$

(a) Specify the region of convergence (ROC) for $H(z)$.

(b) What are the two poles and the two zeros of $H(z)$?

(c) Is the system stable?

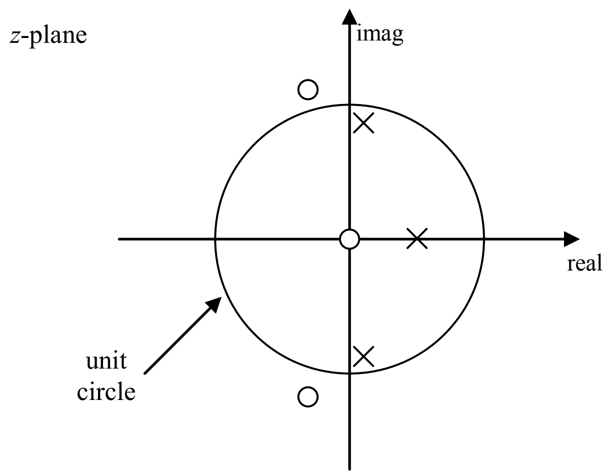
Part II. A real discrete-time signal $x[n]$ is defined

$$x[n] = \left(\frac{1}{4}\right)^n u[n] + 2u[n-2]$$

Determine $X(z)$ and its region of convergence.

Part III.

(a) Determine a simple discrete-time difference equation that has a system function described by the following pole-zero diagram.



zeros: $-0.1 \pm j1.1$; and 0
poles: $0.4 \pm j0.4$, and 0.5

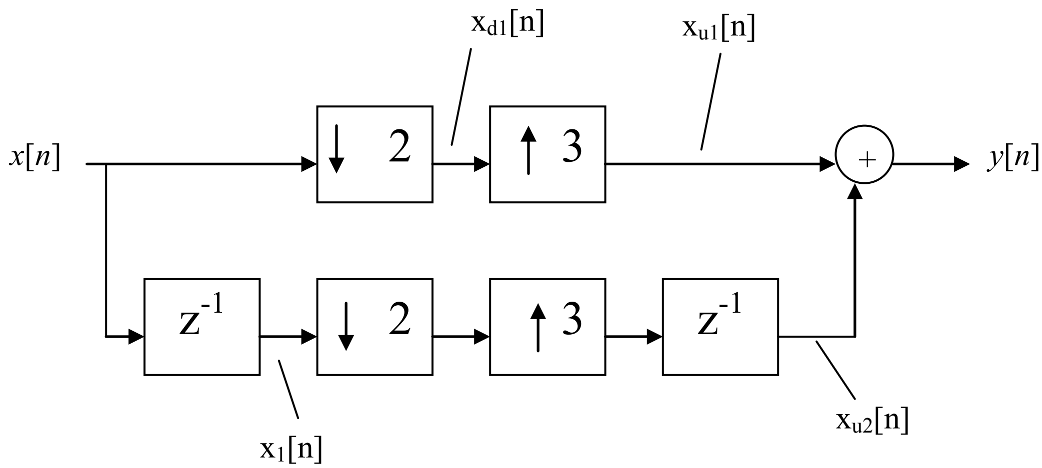
(b) Sketch a block diagram for a *Direct Form II* filter structure that will implement the difference equation.

Part IV. A peculiar upsampling system called the "Reham Processor" takes an input sequence and inserts a zero sample between *pairs* of its input samples. In other words, for an input sequence x[n] the output sequence y[n] is

x[0], x[1], 0, x[2], x[3], 0, x[4], x[5], 0, x[6], x[7], 0, …

Note that no samples are lost in the Reham Processor: we can obtain the original sequence again from y[n].

We can model this system as shown below.



→ Recall that for a downsampling factor *M*, the spectral effect is:

$$X_d\left(e^{j\omega}\right) = \frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j(\omega/M - 2\pi i/M)}\right)$$

→ Recall that for an upsampling factor *L*, the spectral effect is:

$$X_u\left(e^{j\omega}\right) = X\left(e^{j\omega L}\right)$$

Using the formulae for the effects of a delay by z^{-1}, downsampling and upsampling on the signal spectrum, write mathematical expressions *in terms of* the input spectrum $X(e^{j\omega})$ for

$X_1(e^{j\omega})$,

$X_{d1}(e^{j\omega})$,

$X_{u1}(e^{j\omega})$,

and $X_{u2}(e^{j\omega})$